Junior Mathematical Challenge

Solutions 2021

For reasons of space, these solutions are necessarily brief.

There are more in-depth, extended solutions available on the UKMT website, which include some exercises for further investigation:

www.ukmt.org.uk

1. **A** $123 - 456 + 789 = 123 + 789 - 456 = 912 - 456 = 456$.
(*Note that* $123 + 789 = 2 \times 456$.)

2. **E** £20 is equivalent to 2000p and £50 is equivalent to 5000p.
So the number of coins Brianna has is $2000 \div 5 + 5000 \div 2 = 400 + 2500 = 2900$.

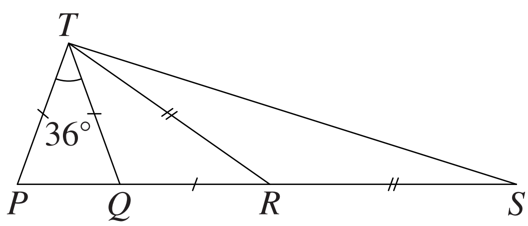
3. **A** $1 - 2 \times 3 + 4 \div 5 = 1 - (2 \times 3) + (4 \div 5) = 1 - 6 + 0.8 = -5 + 0.8 = -4.2$.

4. **D** $187 = 11 \times 17$; $156 = 11 \times 14 + 2$; $253 = 11 \times 23$; $495 = 11 \times 45$; $132 = 11 \times 12$.
So four of the numbers are multiples of 11.
(*Look up the rule for divisibility by 11 to find a quicker method of solving this problem.*)

5. **E** The information given shows that 60% of the distance between my home and school is 1200 metres.
Therefore the full distance, in metres, is $\dfrac{1200}{60} \times 100 = 20 \times 100 = 2000$.

6. **B** $(2 - \frac{1}{2})(3 - \frac{1}{3})(4 - \frac{1}{4}) = \dfrac{3}{2} \times \dfrac{8}{3} \times \dfrac{15}{4} = 15$.

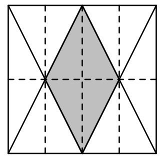
7. **C** In triangle PQT, we know that $PT = QT$, so $\angle TPQ = \angle TQP$.
Now $\angle TQP = \angle QTR + \angle QRT$ (exterior angle theorem) $= 2 \times \angle QTR$ as triangle QTR is isosceles with $QT = QR$.
Therefore, $\angle QTR = \frac{1}{2}\angle TPQ = 72° \div 2 = 36°$.
Similarly, $\angle RTS = \frac{1}{2}\angle QRT = 36° \div 2 = 18°$.
So $\angle PTS = (36 + 36 + 18)° = 90°$.



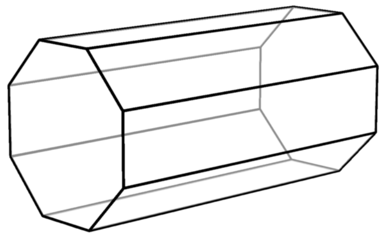
8. **E** $1-(2-(3-(4-5))) = 1-(2-(3-(-1))) = 1-(2-(3+1)) = 1-(2-4) = 1-(-2) = 1+2 = 3$.

9. **E** The solution to 3 across is an odd square. The only two-digit odd squares are 25, 49 and 81. The solution to 1 down is a square, but no squares have a units digit of 2 or 8. So 1 down is 64 and 3 across is 49. Hence $x = 9$. Checking the remainder of the crossnumber, we note that 1 across is a square which has a tens digit of 6 and therefore is 64. Finally, we note that 2 down is now 49, which is indeed a square.

10. C Broken lines have been added to the square so that it has been divided up into sixteen congruent triangles, four of which have been shaded.
So the fraction of the area of the square which has been shaded is $\frac{4}{16} = \frac{1}{4}$.

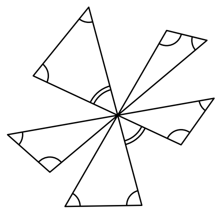


11. C The prism has ten faces, so it has an octagonal cross-section, but not necessarily regular as shown.
It has eight edges at each end and eight edges joining the two ends. Therefore, the number of edges is $3 \times 8 = 24$.



12. D Jasleen answered eight questions every minute, so the number of questions she answered in one hour was $60 \times 8 = 480$.
Ella answered five questions every 40 seconds, so she answered 15 questions every two minutes.
Therefore the time in minutes it took Ella to answer 480 questions was $\frac{480}{15} \times 2 = 32 \times 2 = 64$.
Hence Ella took four minutes longer than Jasleen to answer the questions.

13. B In the diagram, the ten angles which meet at the point of intersection of the five line segments may be divided into five pairs of vertically opposite angles. One such pair of angles has been marked with double arcs. In each pair, one of the angles is in one of the five triangles and one is not.
The sum of the angles which meet at a point is 360°, so the sum of those five angles at the centre which are in triangles is $360° \div 2 = 180°$.
The sum of the interior angles of the five triangles is $5 \times 180° = 900°$.
So the sum of the original marked angles is $(900 - 180)° = 720°$.



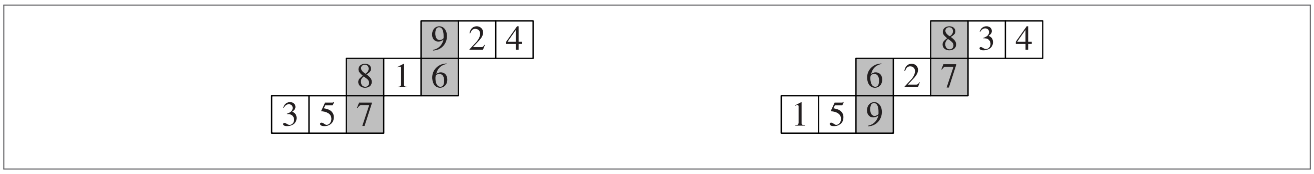
14. A As the result of the calculation described is an integer, the original three-digit positive integer must be a multiple of 9.
Let this integer be x. Then $\frac{x}{9} - 9 \geq 100$. So $\frac{x}{9} \geq 109$. Hence $x \geq 981$.
However, x is a three-digit multiple of 9, so its only possible values are 981, 990 and 999.

15. D After exactly one half of the two pence coins are replaced by ten pence coins, Alex's coins have a mean value of six pence.
Therefore the total number of Alex's coins is $\frac{420}{6} = 70$.
So Alex initially had $70 \times 2\text{p} = £1.40$.

16. B As the cube has edge-length 10 cm, each of the twelve edges has nine dots other than those at a vertex. The cube has eight vertices.
So the total number of dots is $12 \times 9 + 8 = 108 + 8 = 116$.

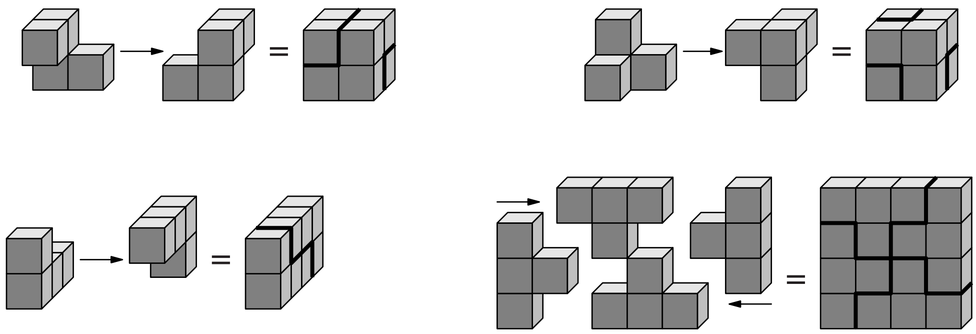
17. C Thirteen of the first fifteen integers may be written as the sum of three squares, as shown below.
$1 = 0^2 + 0^2 + 1^2$; $2 = 0^2 + 1^2 + 1^2$; $3 = 1^2 + 1^2 + 1^2$; $4 = 0^2 + 0^2 + 2^2$; $5 = 0^2 + 1^2 + 2^2$;
$6 = 1^2 + 1^2 + 2^2$; $8 = 0^2 + 2^2 + 2^2$; $9 = 0^2 + 0^2 + 3^2$; $10 = 0^2 + 1^2 + 3^2$; $11 = 1^2 + 1^2 + 3^2$;
$12 = 2^2 + 2^2 + 2^2$; $13 = 0^2 + 2^2 + 3^2$; $14 = 1^2 + 2^2 + 3^2$.
However, it is not possible to find any three-number combination chosen from 0, 1, 4, 9 to sum to 7, or to 15.

18. C The sum of the two numbers in each shaded column is 15. Therefore, one of these columns contains the numbers 7 and 8, while the other column holds the numbers 6 and 9. Note also that two of these numbers, one from each column, combine with the number in the central cell to sum to 15. This is possible only if the number in the central cell is 1 and the shaded numbers in the central row are 6 and 8 or if the number in the central cell is 2 and the shaded numbers in the central row are 6 and 7.
The diagrams below show how the grid may be completed correctly with either 1 or 2 in the central cell, though these are not the only ways of doing so.



19. D Let the number of girls in the class be $3x$. Then the number of girls who study German is x, as is the number of boys who study German. So the number of girls who study French is $2x$ and the number of boys who study French is $2 \times 2x = 4x$.
Therefore the total number of boys and girls in the class is $x + x + 2x + 4x = 8x$ and hence is a multiple of 8. Of the options, only 32 is divisible by 8.

20. E



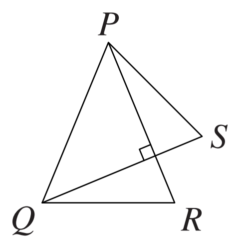
The diagrams show how, for two of the given shapes, two copies of the shape fit together to form a cube of side-length 2. Hence, for each of those shapes, eight copies will fit together to form a $2 \times 4 \times 4$ cuboid. It is also shown that, for the 'L' shape, two copies of the shape will fit together to form a $1 \times 2 \times 4$ cuboid. Hence eight copies of the 'L' shape will fit together to form a $2 \times 4 \times 4$ cuboid. Finally, it is shown that, for the 'T' shape, four copies of the shape will fit together to form a $1 \times 4 \times 4$ cuboid. So eight copies of the 'T' shape will fit together to form a $2 \times 4 \times 4$ cuboid.

21. B Let the numbers of dogs, fish and children in the bay be d, f, c respectively. Then the total number of legs is $4d + 2c$, the number of heads is $d + f + c$ and the number of tails is $d + f$.
Therefore, $4d + 2c = 40 \ldots$ [1] and $d + f + c = 2(d + f) \ldots$ [2].
From [1]: $c = 20 - 2d$. Substituting for c in [2] now gives $d + f + 20 - 2d = 2d + 2f$.
So $3d + f = 20$ and we know $d > f$. Therefore $4d > 20$ (so $d > 5$) and $3d \leq 20$ (so $d \leq 6$).
Therefore $d = 6$ and $f = 20 - 18 = 2$. So there are two fish in the bay.

22. A Let the length and breadth of each of the four rectangles be l cm and b cm respectively.
Then $2l + 2b = 20$, so $l + b = 10$. Note that the four congruent rectangles and the inner square form an outer square, whose side-length is the sum of the length and breadth of one of the rectangles. Hence this outer square has side-length 10 cm and area 100 cm^2.
So the area, in cm^2, of each rectangle is $(100 - 44) \div 4 = 56 \div 4 = 14$.

23. **E** The only four different integers whose product is 9 are $-3, -1, 1, 3$.
Therefore p, q, r and s are 12, 10, 8 and 6 in some order.
So $p + q + r + s = 12 + 10 + 8 + 6 = 36$.

24. **D** Let $\angle PRQ$ be α. Since triangle PQR is isosceles with $PR = PQ$, then $\angle PQR = \angle PRQ = \alpha$.



Also, as PR and QS are perpendicular, $\angle SQR = 90° - \angle PRQ = 90° - \alpha$.
Therefore $\angle PQS = \angle PQR - \angle SQR = \alpha - (90° - \alpha) = 2\alpha - 90°$.
In triangle PQS, it is given that $PQ = QS$.
So $\angle PSQ = \angle QPS = (180° - \angle PQS) \div 2 = (180° - (2\alpha - 90°)) \div 2$
$= (270° - 2\alpha) \div 2 = 135° - \alpha$.
Therefore, the sum of $\angle PRQ$ and $\angle PSQ$ is $\alpha + 135° - \alpha = 135°$.

25. **B** Let the four different integers, in ascending order, be p, q, r, s. Then $p < q < r < s$.
So $p + q < p + r < p + s < q + s < r + s$ and $p + q < p + r < q + r < q + s < r + s$.
Each of these strings of inequalities gives five different values for sums of pairs. So each is the string of five values given in the question: namely $23 < 26 < 29 < 32 < 35$. Hence it may be deduced that $p + q = 23; p + r = 26; p + s = q + r = 29; q + s = 32; r + s = 35$.
Therefore $s - r = (p + s) - (p + r) = 29 - 26 = 3$. So $(r + s) + (s - r) = 35 + 3 = 38$.
Hence $2s = 38$ and the largest of the four integers is 19.
(*It is left as an exercise for the reader to calculate the values of the other three integers and to check that, when added in pairs, the four integers give the sums stated in the question.*)